Omega HW #1 – Calculating functions

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September 21, 2025

Square roots

In class, we talked about two methods of calculating $\sqrt{2}$. In the "decimal" method, we considered the decimal representation $\sqrt{2} = 1.d_1d_2d_3d_4\ldots$ and iteratively calculated d_1, d_2, d_3, and so on. In the "fractional" method, we begin with an initial approximation x (e.g., $x = 1$), and repeated applied the transformation $x \mapsto \frac{1}{2}\left(x + \frac{2}{x}\right)$ to get successively better approximations.

1. (1) Calculate $\sqrt{17}$ to at least 3 decimal places, using the decimal method.
2. (2) Calculate the first three fractional approximations of $\sqrt{17}$. *(Hint: You will need to change the transformation you use.)*
3. Let $f(x) = \frac{1}{2}\left(x + \frac{2}{x}\right)$.
   (a) (1) Calculate $f(\sqrt{2})$.
   (b) (1) Draw a sketch of $y = f(x)$ for $1 \le x \le 2$. You can use a tool to help you, e.g.`https://www.desmos.com/calculator`. Indicate the following points in the graph:
      - The minimum value of the function.
      - The points $(x, f(x))$ for the first three x values in the fractional method (beginning with $x = 1$).

   (c) (2) If x is a number in the interval $[1, 2]$, what can you say about the relative sizes of $|x - \sqrt{2}|$ and $|f(x) - \sqrt{2}|$, based on looking at this graph? Which is larger, and why?

Functions and inverses

Given a function $f(x)$, its *inverse* $f^{-1}(x)$ (if it exists!) is a function such that

- $f^{-1}(f(x)) = x$ for all x in the domain of f, and
- $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.

1. (1) Find the inverse of $f(x) = 4 - \frac{3}{4}x$.
2. (2) Let $f(x) = 3x^2 + 4x + 3$.

- Find a function $g(x)$ such that $f(g(x)) = x$. [1]
- What are the domain and range of $g(x)$?
- Draw the graphs of both $y = f(x)$ and $y = g(x)$.

Bonus question: Cube roots and beyond

1. (2) Think about how you could adapt the decimal method to calculate *cube roots*. Use this method to calculate $\sqrt[3]{2}$ to 3 decimal places. I've included the first step below to get you started. [2]

$$\begin{array}{rl}
 & 1\,.\;\;2 \qquad\qquad \ldots \\ \hline
\big) & 2\,.\,000\,000\,000\ldots \\
- & 1 \\ \hline
 & 1\;\;000 \\
 & -\;\;600 \\
 & -\;\;120 \\
 & -\qquad 8 \\ \hline
 & \quad 272\,000
\end{array}$$

2. (3) Can you describe an algorithm to calculate $\sqrt[n]{x}$ for any whole number n? What about $x^{m/n}$ for any whole numbers m and n? (Remember that fractional exponents are defined by $x^{m/n} = (\sqrt[n]{x})^m$.)

[1] *Hint:* Complete the square.

[2] Doing this by hand is computation-heavy, and requires multiplying 3-digit integers along the way. You can use a calculator to help you.