MATH 220.201 CLASS 9 QUESTIONS

1. Prove that the equation $x^6 + x^4 + 2x^2 + 1 = 0$ has no real solutions.

2. Let x be a nonzero real number. If $x + \frac{1}{x} < 2$, then $x < 0$.[1]

3. Let x be an irrational number. Then there is no **largest** rational number y with the following property: $y \leq x$.

Use the following theorem in questions 4 and 5:

Intermediate Value Theorem: For every continuous function f on the closed interval $[a, b]$, and for every number k between $f(a)$ and $f(b)$, there is some $c \in [a, b]$ such that $f(c) = k$.

4. • The equation $x^5 + 2x - 5 = 0$ has a solution on the interval $[1, 2]$.

   • The equation $x^5 + 2x - 5 = 0$ has *exactly one* solution on the interval $[1, 2]$.

5. Any polynomial equation $f(x) = 0$ of odd degree has a real number solution.

[1] You can adapt your argument to prove the following well-known theorem.

Arithmetic Mean - Geometric Mean Inequality: For any positive real numbers x and y, $\sqrt{xy} \leq \frac{x+y}{2}$.