MATH 220.201 CLASS 7 QUESTIONS

Today we'll revisit some basics of set theory. These statements will be simple but it will give us practice with writing proofs. It is also an opportunity to solidify our fundamentals. We can assume all the axioms of logic (i.e. Chapter 2).

1. Let A, B be sets. Prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

2. Let A, B be sets. Prove that $A = (A - B) \cup (A \cap B)$. (Hint: Let $x \in A$. Case 1: $x \in B$. Case 2: $x \notin B$.)

3. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

4. Let A, B be sets.
   (a) Prove that if $A = \emptyset$ or $B = \emptyset$, then $A \times B = \emptyset$.

   (b) Prove that if $A \times B = \emptyset$, then $A = \emptyset$ or $B = \emptyset$.

5. Let A, B, C, D be sets.
   (a) Prove that if $A \subseteq C$ and $B \subseteq D$, then $(A \times B) \cap (C \times D) = A \times D$.

   (b) Prove that if $(A \times B) \cap (C \times D) = A \times D$ then $A \subseteq C$ and $B \subseteq D$.