MATH 220.201 CLASS 6 QUESTIONS

1. Let $n \in \mathbb{Z}$. Prove or disprove: n is odd if and only if $4n^3 - 2n + 1$ is odd.

   Definition 0.1 (Divisibility)**.** *Let $a, b \in \mathbb{Z}$. a* **divides** *b (written $a|b$) if there is some $n \in \mathbb{Z}$ such that $b = an$. Here are some properties you can assume. They are a good warmup if you want practice with proofs.*
   - $a|b \wedge b|c \implies a|c$
   - $a|c \wedge b|d \implies ab|cd$
   - $a|b \wedge a|c \implies a|(bx + cy)$
   - $\forall a \in \mathbb{Z}, a|0$
   - $\forall a \in \mathbb{Z}, 1|a$

2. Prove or find a counterexample: For all $a, b \in \mathbb{Z}$, if $3|ab$, then $(3|a$ or $3|b)$.
3. Prove or find a counterexample: For all $a, b \in \mathbb{Z}$, if $4|ab$, then $(4|a$ or $4|b)$.

   Definition 0.2 (Congruence)**.** *Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. a is* **congruent** *to b* **modulo** *n if n divides $a - b$. We write this as*

   $$a \equiv b \pmod{n}$$

   Here are some properties you can assume.
   - $a \equiv b \pmod{n} \implies a + c \equiv b + c \pmod{n}$ *and* $ac \equiv bc \pmod{n}$
   - $\exists r \in \{0, 1, 2, \ldots, n-1\}, a \equiv r \pmod{n}$

4. Prove or disprove: For all $n \in \mathbb{Z}$, $3|n$ or $n^2 \equiv 1 \pmod{3}$.
5. Prove or disprove: For all $n \in \mathbb{Z}$,

   $$((2 \nmid n) \wedge (3 \nmid n)) \implies \exists m \in \mathbb{Z}, mn \equiv 1 \pmod{6}$$

6. Prove or disprove: For all $n \in \mathbb{Z}$,

   $$n^3 \not\equiv 1 \pmod{7} \implies (n^3 \equiv 1 \pmod{7}) \vee (n \equiv 0 \pmod{7})$$

7. Prove: For all $n \in \mathbb{Z}$,

   $$n \equiv 3 \pmod{4} \implies \sim (\exists a, b \in \mathbb{Z}, a^2 + b^2 = n)$$

   Definition 0.3 (Relatively prime)**.** *Let $a, b \in \mathbb{Z}$. a and b are* **relatively prime** *(written $\gcd(a, b) = 1$, or just $(a, b) = 1$) if*

   $$\forall n \in \mathbb{N}, (n|a \implies n \nmid b)$$

8. Prove that 5 and 12 are relatively prime.
9. Prove that if $a \equiv 7 \pmod{10}$, then a and 10 are relatively prime.