MATH 220.201 CLASS 5 QUESTIONS

1. Let $n \in \mathbb{Z}$. Prove that if n is odd, then $n^2 - 5n + 2$ is even.
2. Let $n \in \mathbb{Z}$. Prove that n is odd if and only if n^2 is odd.
3. Let $n \in \mathbb{Z}$. Prove that if $7n + 4$ is even, then $3n - 11$ is odd.
4. Suppose that the following fact is known to be true

   Lemma 0.1. *For every $k \in \mathbb{Z}$, $k(k+1)$ is an even integer.*

   Prove that if n is any odd integer, then $n^2 - 1$ is a multiple of 8.
5. Let $n \in \mathbb{Z}$. Prove that $n^2 - 3n + 9$ is odd.
6. Let $a, b \in \mathbb{Z}$. Prove that

$$ab \text{ is even} \iff (a \text{ is even}) \vee (b \text{ is even})$$

7. The following is a faulty proof. Explain what is wrong with it.

Proposition 0.2. *If m is an even integer and n an odd integer, then $3m+n+mn-1$ is a multiple of 4.*

Proof. Let m be an even integer and n an odd integer. Then $m = 2k$ and $n = 2k+1$ for some $k \in \mathbb{Z}$. Therefore

$$\begin{aligned} 3m+n+mn-1 &= 3(2k)+(2k+1)+2k(2k+1)-1 \\ &= 6k+2k+1+4k^2+2k-1 \\ &= 4k^2+8k \\ &= 4(k^2+2k) \end{aligned} \tag{0.1}$$

Since k^2+2k is an integer, $3m+n+mn-1$ is a multiple of 4. □

8. Let $a,b,c,d,n \in \mathbb{Z}$. Prove that if $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$, where the notation '$\equiv \pmod{n}$' is defined below.

Definition 0.3. *For integers a, b, n, if $a-b$ is a multiple of n, let us write this as*

$$n|(a-b) \text{ ; 'n divides } a-b\text{', or}$$

$$a \equiv b \pmod{n} \text{ ; '}a\text{ is congruent to } b \text{ modulo } n\text{'.}$$