MATH 220.201 CLASS 4 QUESTIONS

(1) Show that $\sim (P \iff Q) \equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$.

(2) Let $P(n)$ be the open sentence
$$P(n): \frac{5n - 3}{6} \text{ is an integer}$$
over the domain $n \in \mathbb{Z}$.
  (a) Is the following true or false? $\exists n \in \mathbb{Z}, P(n)$. Explain.
  (b) Is the following true or false? $\forall n \in \mathbb{Z}, P(n)$. Explain.

(3) Rewrite the following using quantifiers:
  (a) There is a real number such that $1/y = y + 1$.
  (b) For every natural number z, there is a natural number w such that $z^2 < w$.

(4) Let A, B be subsets of a universal set U. Rewrite the following using quantifiers involving the elements. For example, $\forall x \in U, \ldots$
  (a) $A \subset B$
  (b) $A \cap B = \emptyset$
  (c) $A \not\subset B$

(5) Is the following statement true or false?
$$\forall \epsilon \in \mathbb{R}_{>0}, \exists \delta \in \mathbb{R}_{>0}, |(3 + \delta)^2 - 3^2| < \epsilon$$
Write its negation.

(6) For what values of x is the following open sentence true?
$$\forall \epsilon \in \mathbb{R}_{>0}, \exists \delta \in \mathbb{R}_{>0}, |(x + \delta)^2 - x^2| < \epsilon$$
How about this one?
$$\exists \delta \in \mathbb{R}_{>0}, \forall \epsilon \in \mathbb{R}_{>0}, |(x + \delta)^2 - x^2| < \epsilon$$