MATH 220.201 CLASS 3 QUESTIONS - SOLUTIONS

(1) For each of the following sentences, say whether it is a *statement* or an *open sentence*. Can you state its negation?
   (a) 5 is even or 3 is prime.
   Solution: It is a statement, and its negation is '5 is not even and 3 is not prime.'
   (b) At least one of my two friends misplaced his/her homework assignment.
   Solution: It is a statement, and its negation is 'Neither of my two friends misplaced their homework assignment.'
   (c) For any polyhedron, the number of vertices plus the number of faces equals the number of edges plus 2.
   Solution: It is a statement, and its negation is 'There is a polyhedron such that the number of vertices plus the number of faces is not equal to the number of edges plus 2.'
   (d) If $x^4 = 1$, then $x = 1$ or $x = -1$.
   Solution: It is an open sentence, because there is an unquantified variable.

(2) Construct a truth table in P, Q for the compound statement $(P \vee Q) \wedge \sim (P \wedge Q)$.[1]

   Solution: The values in the truth table are F, T, T, F.

(3) Construct a truth table in P, Q for the statement $P \implies Q$. Can you construct a compound statement with the same truth table using only $\sim, \vee$, and $\wedge$?

   Solution: The truth table is the same as that of $\sim P \vee Q$.

(4) Let $A = \{3, 6, 8, 9, 11\}$ and $B = \{6, 9, 10\}$. Find all sets S of integers such that the following statement holds true for all integers x.

$$(x \in S) \implies (x \in A) \wedge (x \in B)$$

   Is there a set S such that $(x \in S) \iff (x \in A) \wedge (x \in B)$?

   Solution: S satisfies the given open sentence for all integers x iff $S \in \mathcal{P}(A \cap B)$, i.e. $S \subseteq A \cap B = \{6, 9\}$. That is,

$$S = \emptyset, \{6\}, \{9\}, \{6, 9\}$$

   satisfy the property $\forall x \in \mathbb{Z}, (x \in S) \implies (x \in A) \wedge (x \in B)$. When $S = \{6, 9\}$, we have

$$\forall x \in \mathbb{Z}, (x \in S) \iff (x \in A) \wedge (x \in B)$$

[1] This is sometimes called 'exclusive or', or 'xor'.