

## MATH 220.201 CLASS 2 QUESTIONS

### 1. COMPLEMENT AND SET DIFFERENCE

Let  $U$  denote the universal set.

- (1) Prove that  $A - B = A \cap \overline{B}$ .
- (2) What is  $\overline{\overline{U}}$ ?
- (3) (De Morgan's Laws) Prove that  $\overline{A \cup B} = \overline{A} \cap \overline{B}$  and  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ .
- (4) Find three sets  $A, B$ , and  $C$  such that  $(A \cup B) \cap C \neq A \cup (B \cap C)$ . (Hint: Draw Venn diagrams for  $(A \cup B) \cap C$  and  $A \cup (B \cap C)$ .)

### 2. INDEXED UNION AND INTERSECTION

- (1) For each  $k \in \mathbb{N}$ , define  $A_k \subset \mathbb{R}$  by  $A_k = [\frac{1}{k+1}, \frac{1}{k}]$ . What is  $\bigcup_{k=1}^{\infty} A_k$ ?
- (2) For each  $k \in \mathbb{N}$ , define  $B_k \subset \mathbb{R}$  by  $B_k = (-\frac{1}{k}, \frac{1}{k})$ . What is  $\bigcap_{k=1}^{\infty} B_k$ ?
- (3) Let  $A$  and  $B$  be sets. Prove that  $\bigcap_{b \in B} (A - \{b\}) = A - B$ .

### 3. SET PARTITIONS AND CARTESIAN PRODUCT

- (1) List out the partitions of the set  $\{1, 2, 3\}$ . How many partitions are there for the set  $\{1, 2, 3, 4\}$ ? (Can you count them without listing them out?)
- (2) Construct a partition of  $\mathbb{Z}$  into two sets.
- (3) Construct a partition of  $\mathbb{Z}$  into three *infinite* sets.
- (4) Construct a partition of  $\mathbb{Q}$  into two infinite sets.
- (5) How many elements are in the set  $\{(x, y) | (x, y) \in \mathbb{R}^2 \text{ and } x^2 + y^2 < 12\}$ ?
- (6) Let  $S \subset \mathbb{R}^2$  be the set of points shown on the left. Write down, in terms of  $S$ , the set on the right (i.e., a reflection over the line  $y = \frac{1}{2}$ ).

