

## MATH 220.201 CLASS 25 QUESTIONS

1. Prove that the sequence  $\left\{ \frac{4n^2-3}{5n^2-2n} \right\}$  converges to  $\frac{4}{5}$ .
2. (12.11) Prove (using the definition of convergence) that if a sequence  $\{s_n\}$  converges to  $L$ , then the sequence  $\{s_{n^2}\}$  converges to  $L$ .
3. (Adapted from 2011 WT2 Q7) Let  $\{b_n\}$  be a sequence defined by
$$b_1 = 2 \text{ and } b_{n+1} = \frac{b_n + \sqrt{b_n}}{2}$$
  - (a) Prove that  $1 < b_{n+1} < b_n$  for every  $n \in \mathbb{N}$ .
  - (b) Prove that  $\{b_n\}$  converges to 1.

4. (2013 WT1, Q8) Prove that the function  $f : (-1, 1] - \{0\} \rightarrow \mathbb{R}$  given by  $f(x) = x - \frac{1}{x}$  is a bijection.

5. Give an example of

- A function  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  which is injective, but not surjective.
- A function  $g : \mathbb{Z} \rightarrow \mathbb{Z}$  which is surjective, but not injective.
- Prove that for any such functions  $f$  and  $g$ ,  $f \circ g$  cannot possibly be bijective.
- Give such examples with the property that  $g \circ f$  is bijective.