MATH 220.201 CLASS 23 QUESTIONS

Definition 0.1. *Let* $\{a_n\} = a_1, a_2, a_3, \ldots$ *be a sequence of real numbers, and let* L *be another real number.* $\{a_n\}$ *is said to* **converge to** L *if the following holds:*

For every real number $\epsilon > 0$*, there exists some* $N \in \mathbb{N}$*, such that for every natural number* $n > N$*,* $|a_n - L| < \epsilon$*.*

1. Prove that the sequence $\{\frac{n}{2n+1}\}$ converges.

2. Prove that the sequence $\{n\}$ diverges.

3. Does the sequence $\{\frac{2^n-1}{2^n} + \frac{(-1)^n}{n^2}\}$ converge?

4. Suppose that $\{a_n\}$ is a sequence of real numbers which converges to L, and suppose that $a_n > 1$ for every n. Is it true that $\{1/a_n\}$ converges to $1/L$?

5. Prove that the sequence $\{a_n\}$ defined by $a_1 = 1$ and $a_{n+1} = a_n + \frac{1}{a_n}$ diverges. (Hint: Suppose that it converges, and use proof by contradiction.)

6. Prove that the sequence $\{a_n\}$ defined by $a_1 = 1$ and $a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}$ converges to $\sqrt{2}$. (Hint: Define a new sequence $\{b_n\}$ by $b_n = a_n - \sqrt{2}$. Now find a recursive formula for this new sequence and use that to show that it converges to 0.)