MATH 220.201 CLASS 22 QUESTIONS

1. Let A and B be sets, and suppose there is an injective function $f : A \to B$. Can you construct a surjective function $g : B \to A$?

2. Let A and B be sets, and suppose there is a surjective function $g : B \to A$. Can you construct an injective function $f : A \to B$?

3. Prove that
$$|2^{\mathbb{N}}| = |2^{\mathbb{N}} \times 2^{\mathbb{N}}|$$
by finding an explicit bijection. (Remember that 2^A is the set of functions from A to $\{0, 1\}$.)

4. Let A, B, and C be any three sets, and let A, B be disjoint. Prove that $|C^{A\cup B}| = |C^A \times C^B|$, or written another way,
$$|\mathrm{Fun}(A \cup B, C)| = |\mathrm{Fun}(A, C) \times \mathrm{Fun}(B, C)|$$

5. (More challenging) Prove that $|C^{A\times B}| = |(C^A)^B|$, or written another way,
$$|\mathrm{Fun}(A \times B, C)| = |\mathrm{Fun}(B, \mathrm{Fun}(A, C))|$$