MATH 220.201 CLASS 20 QUESTIONS

1. (a) Let $\mathbb{Q}^+$ denote the set of positive rational numbers. Prove that there exists a surjective function $\mathbb{N} \times \mathbb{N} \to \mathbb{Q}^+$.

    (b) Deduce that there exists a surjective function $\mathbb{Z} \times \mathbb{N} \to \mathbb{Q}$.

    (c) Show that there is an injective function $\mathbb{Q} \to \mathbb{Z} \times \mathbb{N}$.

2. For any integer k, let $k^{\mathbb{N}}$ denote the set of functions $\mathbb{N} \to \{1, 2, \ldots, k\}$. We'll prove in class today that $2^{\mathbb{N}}$ is *uncountable*.
    (a) Explain why there is a bijection between $2^{\mathbb{N}}$ and $\mathcal{P}(N)$, the *power set* of $\mathbb{N}$.

(b) Show that there is a surjective function $10^{\mathbb{N}} \to [0,1]$, where $[0,1]$ is the interval of real numbers x such that $0 \le x \le 1$.

(c) Show that, for any positive integer k, there is a surjective function $k^{\mathbb{N}} \to [0,1]$.

3. Let $S \subset \mathcal{P}(\mathbb{N})$ be the set of *finite* subsets of $\mathbb{N}$. Prove that S is denumerable.