

MATH 220.201 CLASS 19 QUESTIONS

1. Let $f : A \rightarrow B$ and $g : C \rightarrow D$ be functions. Then one can form the function

$$f \times g : A \times C \rightarrow B \times D$$

- (a) Prove that if f is injective and g is injective, then $f \times g$ is injective.
- (b) The same if both are surjective.
- (c) Come up with an example where f is injective, g is surjective, and $f \times g$ is neither.
- (d) How about an example where f is not bijective, but $f \times g$ is? (Hint: use \emptyset .)

2. Let A, B , and C be sets. Suppose that $g : A \rightarrow B$, $h : A \rightarrow B$, and $f : B \rightarrow C$ are functions with the property that $f \circ g = f \circ h$.

- (a) Provide an example of the situation above which shows that g does not necessarily have to equal h .
- (b) Prove that if f is injective, then $g = h$.