## MATH 220.201 CLASS 18 QUESTIONS

1. For each of the following pairs of sets $A, B$, determine whether there are functions from $A$ to $B$ which are one-to-one (injective), onto (surjective), or both (bijective). Do the same with functions from $B$ to $A$.
(a) $A=\{1,2,3,4,5\}$ and $B=\{6,7,8,9\}$.
(b) $A=\mathbb{N}=\{1,2,3, \ldots\}$ and $B=\{2 n: n \in \mathbb{N}\}=\{2,4,6,8, \ldots\}$.
(c) $A=\mathbb{N}$ and $B=\{a+b \sqrt{2}: a \in \mathbb{N}, b \in\{0,1,2\}\}$.
(d) $A=\mathbb{N}$ and $B=\mathbb{Z}=\{\ldots,-2,-1,0,1,2, \ldots\}$.
(e) $A=\mathbb{N}, B=\{2,3,5,7,11, \ldots\}$ is the set of prime numbers.
2. Can you come up with a rigorous definition of what it means for a set to have 'size $n$ '?
3. What about what it means for a set to be 'infinite'?
4. Let $m$ and $n$ be two positive integers such that $m \leq n$, and suppose that $S$ is a set and there's an injection $\{1, \ldots, n\} \rightarrow S$. Prove that if there is an injection $S \rightarrow\{1, \ldots, m\}$, then $m=n$.
5. Let $S$ be a set and suppose that there is a bijection $f: \mathbb{N} \rightarrow S$. Prove that if $T$ is any infinite subset of $S$, then there is a bijection $S \rightarrow T$.
