MATH 220.201 CLASS 18 QUESTIONS

- 1. For each of the following pairs of sets A, B, determine whether there are functions from A to B which are one-to-one (injective), onto (surjective), or both (bijective). Do the same with functions from B to A.
 - (a) $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7, 8, 9\}$.
 - (b) $A = \mathbb{N} = \{1, 2, 3, \ldots\}$ and $B = \{2n : n \in \mathbb{N}\} = \{2, 4, 6, 8, \ldots\}.$
 - (c) $A = \mathbb{N}$ and $B = \{a + b\sqrt{2} : a \in \mathbb{N}, b \in \{0, 1, 2\}\}.$
 - (d) $A = \mathbb{N}$ and $B = \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}.$
 - (e) $A = \mathbb{N}, B = \{2, 3, 5, 7, 11, ...\}$ is the set of prime numbers.
- 2. Can you come up with a rigorous definition of what it means for a set to have 'size n'?
- 3. What about what it means for a set to be 'infinite'?

4. Let m and n be two positive integers such that $m \leq n$, and suppose that S is a set and there's an injection $\{1, \ldots, n\} \to S$. Prove that if there is an injection $S \to \{1, \ldots, m\}$, then m = n.

5. Let S be a set and suppose that there is a bijection $f : \mathbb{N} \to S$. Prove that if T is any infinite subset of S, then there is a bijection $S \to T$.