

MATH 220.201 CLASS 16 QUESTIONS

1. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{\pi, e, \sqrt{2}\}$. Define a function $f : A \rightarrow B$ by

$$f(1) = f(2) = f(4) = \pi \quad f(5) = e \quad f(3) = f(6) = \sqrt{2}$$

Give a relation \mathcal{R} on A with the property that $a\mathcal{R}b \iff f(a) = f(b)$. That is, list out the elements of \mathcal{R} .

2. Prove the following statement: For an equivalence relation \mathcal{R} on A , $a\mathcal{R}b \iff [a]_{\mathcal{R}} = [b]_{\mathcal{R}}$.

3. Let \mathcal{R} be an equivalence relation on A , and consider the function $f : A \rightarrow A/\mathcal{R}$ defined by $f(a) = [a]_{\mathcal{R}}$. Suppose that f is injective. Then what can you say about \mathcal{R} ?

4. Consider the function $f : \mathbb{Z}_5 \rightarrow \mathbb{Z}_5$ given by $f([x]) = [3x + 1]$. Prove that f is both injective and surjective. (Note: you only need to prove one of injectivity and surjectivity to deduce the other. Why?)

