## MATH 220.201 CLASS 16 QUESTIONS

1. Let  $A = \{1, 2, 3, 4, 5, 6\}$  and  $B = \{\pi, e, \sqrt{2}\}$ . Define a function  $f : A \to B$  by  $f(1) = f(2) = f(4) = \pi$  f(5) = e  $f(3) = f(6) = \sqrt{2}$ Give a relation  $\mathcal{R}$  on A with the property that  $a\mathcal{R}b \iff f(a) = f(b)$ . That is,

Give a relation  $\mathcal{R}$  on A with the property that  $a\mathcal{R}b \iff f(a) = f(b)$ . That is, list out the elements of  $\mathcal{R}$ .

2. Prove the following statement: For an equivalence relation  $\mathcal{R}$  on A,  $a\mathcal{R}b \iff [a]_{\mathcal{R}} = [b]_{\mathcal{R}}$ .

- 3. Let  $\mathcal{R}$  be an equivalence relation on A, and consider the function  $f : A \to A/\mathcal{R}$  defined by  $f(a) = [a]_{\mathcal{R}}$ . Suppose that f is injective. Then what can you say about  $\mathcal{R}$ ?
- 4. Consider the function  $f : \mathbb{Z}_5 \to \mathbb{Z}_5$  given by f([x]) = [3x + 1]. Prove that f is both injective and surjective. (Note: you only need to prove one of injectivity and surjectivity to deduce the other. Why?)

5. Let  $A = \mathbb{R}$ , and consider the equivalence relation  $\mathcal{R}$  on A given by  $a\mathcal{R}b$  iff  $b - a \in \mathbb{Z}$ . How many equivalence classes are there? Can you describe them?

6. Let A be the set of lines in  $\mathbb{R}^2$ , and consider the equivalence relation where two lines are equivalent if they are either the same line, or are parallel. How many equivalence classes are there? Can you describe them?