## MATH 220.201 CLASS 16 QUESTIONS

1. Let $A=\{1,2,3,4,5,6\}$ and $B=\{\pi, e, \sqrt{2}\}$. Define a function $f: A \rightarrow B$ by

$$
f(1)=f(2)=f(4)=\pi \quad f(5)=e \quad f(3)=f(6)=\sqrt{2}
$$

Give a relation $\mathcal{R}$ on $A$ with the property that $a \mathcal{R} b \Longleftrightarrow f(a)=f(b)$. That is, list out the elements of $\mathcal{R}$.
2. Prove the following statement: For an equivalence relation $\mathcal{R}$ on $A, a \mathcal{R} b \Longleftrightarrow$ $[a]_{\mathcal{R}}=[b]_{\mathcal{R}}$.
3. Let $\mathcal{R}$ be an equivalence relation on $A$, and consider the function $f: A \rightarrow A / \mathcal{R}$ defined by $f(a)=[a]_{\mathcal{R}}$. Suppose that $f$ is injective. Then what can you say about $\mathcal{R}$ ?
4. Consider the function $f: \mathbb{Z}_{5} \rightarrow \mathbb{Z}_{5}$ given by $f([x])=[3 x+1]$. Prove that $f$ is both injective and surjective. (Note: you only need to prove one of injectivity and surjectivity to deduce the other. Why?)
5. Let $A=\mathbb{R}$, and consider the equivalence relation $\mathcal{R}$ on $A$ given by $a \mathcal{R} b$ iff $b-a \in \mathbb{Z}$. How many equivalence classes are there? Can you describe them?
6. Let $A$ be the set of lines in $\mathbb{R}^{2}$, and consider the equivalence relation where two lines are equivalent if they are either the same line, or are parallel. How many equivalence classes are there? Can you describe them?

