MATH 220.201 CLASS 15 QUESTIONS

Remember: if A and B are sets, then a *relation* from A to B is a subset $R \subseteq A \times B$.

A *function* is a subset $f \subseteq A \times B$ such that for each $a \in A$, there is *exactly one* $b \in B$ such that $(a, b) \in f$. We also use the notation $f(a) = b$, and write $f : A \to B$ to say 'f is a function from A to B. The subset $f \subseteq A \times B$ is called the *graph* of the function.

A is called the *domain* of f.

B is called the *codomain* of f.

The *image* of a set $C \subseteq A$ (written $f(C)$) is the set $\{b \in B | \exists c \in C, f(c) = b\}$.

The *range* of f is $f(A)$.

The *preimage* of a set $D \subseteq B$ (written $f^{-1}(D)$) is the set $\{a \in A | f(a) \in D\}$.

(1) Prove that the function $f : [0, \infty) \to \mathbb{R}$ defined by $f(x) = x^2$ is injective, but not surjective.

(2) Prove that the function $f : \mathbb{R} \to [0, \infty)$ defined by $f(x) = x^2$ is surjective, but not injective.

(3) Prove that the function $f : \mathbb{R} \to (0, \infty)$ defined by $f(x) = 2^x$ is injective. (You can also prove that it is surjective without using logarithms, but it involves the Intermediate Value Theorem.)

(4) Remember that for any positive integer n, $\mathbb{Z}_n = \{[0], [1], \ldots, [n-1]\}$ is the set of *equivalence classes* for the equivalence relation '$\equiv \pmod{n}$'.

  (a) Prove that the function $f : \mathbb{Z} \to \mathbb{Z}_5$ defined by $f(x) = [x]$ is surjective, but not injective. (This isn't a trick question, it is just to check that you understand the definitions.

  (b) Prove that the function $f : \mathbb{Z}_5 \to \mathbb{Z}_5$ defined by $f([x]) = [3x+1]$ is a bijection.