MATH 220.201 CLASS 14 QUESTIONS

(1) For each of the following relations f from A and B, is it a function? If it is, write an expression for $f(a)$ in terms of a. If not, explain why.

   (a) $A = B = \{1, 2, 3, 4\}$, $f = \{(1, 2), (2, 3), (1, 3), (4, 4)\}$.

   (b) $A = \mathbb{R} - \{1\}$, $B = \mathbb{R}$, $f = \{(a, b) | \frac{1}{a-1} = b\}$.

   (c) $A = B = \mathbb{N}$, $f = \{(2n - 1, n) | n \in \mathbb{N}\} \cup \{(2n, n) | n \in \mathbb{N}\}$.

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^2$. Determine the following sets.

   (a) $f([0, 4])$

   (c) $f^{-1}([0, 9])$

   (b) $f([-1, 2])$

   (d) $f^{-1}([1, 4])$

(3) Suppose that A, B are sets and $f : A \to B$ is a function.

   (a) If $C \subseteq A$, is it necessarily true that $f^{-1}(f(C)) = C$?

   (b) If $D \subseteq B$, is it necessarily true that $f(f^{-1}(D)) = D$?