MATH 220.201 CLASS 13 QUESTIONS

(1) $S = \mathbb{Z}$; $a \sim b$ if $a \mid b$. This is not an equivalence relation. It is reflexive and transitive, but it is not symmetric. For example, $2 \sim 6$ but $6 \not\sim 2$.

(2) $S = \mathbb{Z}$; $a \sim b$ if either $a \nmid b$ or $b \nmid a$. This is not an equivalence relation, because it is not reflexive. $n \not\sim n$ for every n. (In fact, this relation is exactly $a \sim b$ if $a \neq b$.)

(3) $S = \mathbb{Z}$; $a \sim b$ if $a \equiv b \pmod 6$. This is an equivalence relation.

(4) $S = \mathbb{N}$; $a \sim b$ if a and b have a common prime factor. This is not an equivalence relation. It is reflexive and symmetric, but it is not transitive. $4 \sim 6$ and $6 \sim 9$ but $4 \not\sim 9$.

(5) $S = \mathbb{R}$; $a \sim b$ if $b - a \in \mathbb{Z}$. This is an equivalence relation.

(6) $S = \mathbb{R}$; $a \sim b$ if $|b - a| < 1$. This is not an equivalence relation. It is reflexive and symmetric, but it is not transitive. For example, $0 \sim \frac{2}{3}$ and $\frac{2}{3} \sim \frac{4}{3}$ but $0 \not\sim \frac{4}{3}$.

(7) $S = \mathbb{R}$; $a \sim b$ if $\sqrt{e^a + 2} = \sqrt{e^b + 2}$. This is an equivalence relation. In fact, for any function $f(x)$, the relation $a \sim b \iff f(a) = f(b)$ is an equivalence relation.

(8) $S = \mathbb{Z}$; $a \sim b$ if $3a + 5b$ is even. This is an equivalence relation. It is equivalent to saying $a \equiv b \pmod 2$.

(9) $S = \mathbb{Z}$; $a \sim b$ if $a + b$ is odd. This is not an equivalence relation, because it is not reflexive or transitive. It is equivalent to saying that a and b have opposite parity.

(10) S is the set of lines in the plane; $\ell_1 \sim \ell_2$ if either $\ell_1 = \ell_2$ or $\ell_1 \mid\mid \ell_2$. This is an equivalence relation.

(11) S is the set of lines in the plane; $\ell_1 \sim \ell_2$ if either $\ell_1 = \ell_2$ or $\ell_1 \perp \ell_2$. This is not an equivalence relation. It is reflexive and symmetric, but not transitive. For example, the line $x = 0$ is perpendicular to the line $y = 0$, which is equivalent to the line $x = 1$, but the lines $x = 0$ and $x = 1$ are not equivalent.

(12) S is the set of lines in the plane; $\ell_1 \sim \ell_2$ if $\ell_1 = \ell_2$ or $\ell_1 \perp \ell_2$ or $\ell_1 \mid\mid \ell_2$. This is an equivalence relation.

(13) S is the set of lines in $\mathbb{R}^3$ containing $(0, 0, 0)$; $\ell_1 \sim \ell_2$ if $\ell_1 = \ell_2$ or $\ell_1 \perp \ell_2$. This is not an equivalence relation. It is reflexive and symmetric, but not transitive. The lines going through the points $(0, 1, 0)$ and $(0, 1, 1)$ are both orthogonal to the line going through $(1, 0, 0)$ but these are not orthogonal to each other.