MATH 220.201 CLASS 12 QUESTIONS

1. Let $a_1, a_2, \ldots$ be a sequence defined by $a_1 = 2$, $a_2 = 1$, and
$$a_{n+1} = a_n + 6a_{n-1}$$
for $n \geq 2$. Prove that, for all n, $a_n = 3^{n-1} + (-2)^{n-1}$.

2. Let $a_1, a_2, \ldots$ be a sequence defined by $a_1 = 1$, $a_2 = 2$, and
$$a_{n+1} = a_n + a_{n-1} + 1$$
for all $n \geq 2$. Conjecture a formula for a_n and then prove your formula.

3. For any positive integer, n is called *prime* if $n \geq 2$ and there exist no integers a such that $1 < a < n$ and $a|n$. Prime numbers are usually denoted by the letter p.

   Prove that any integer $n \geq 2$ is either prime or can be written as a product of (not necessarily distinct) primes.

4. (Binary Representation) Prove that any positive integer n can be written as
$$n = 2^{i_1} + 2^{i_2} + \ldots + 2^{i_k}$$
for some integers $i_1, \ldots, i_k$ with the property that $0 \leq i_1 < i_2 < \cdots < i_k$. (You may assume the fact that for any positive integer n, there is a unique greatest integer i such that $2^i \leq n$.)

   Can you prove this representation is *unique*?