MATH 220.201 CLASS 11 QUESTIONS

1. Prove that if $n \geq 2$ is a natural number and $A_1, A_2, \ldots, A_n$ are sets, then

$$\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \overline{A_1} \cap \overline{A_2} \cap \cdots \cap \overline{A_n}$$

2. Prove that for every odd integer $n \geq 5$, $2^n > n^2$.

3. Prove that for every odd integer n, $5 | 4^n + 1$.

4. Prove that every natural number $n \geq 8$ can be written in the form $n = 5a + 3b$ where a, b are nonnegative integers.

5. Consider the following statement.

For every integer $k \geq 5$, there exists a natural number N such that for every integer $n \geq N$, $2^n > n^k$.

(a) What is wrong with the following argument disproving the statement?

Proof. Suppose there is such an N. Then for every $k \geq 5$ and every $n \geq N$, we have $2^n > n^k$. This means $\log_n(2^n) > k$. But this doesn't hold when $k \geq \log_n(2^n)$. This is a contradiction! $\square$

(b) I think there is a better way to name the variables:

For every integer $k \geq 5$, there exists a natural number N_k such that for every integer $n \geq N_k$, $2^n > n^k$.

Why is this better?

(c) Can you prove the statement? (This is challenging and may require multiple steps! Hint: base case $N_k = 2^k$.)