

MATH 220.201 CLASS 10 QUESTIONS

Use induction to prove the following results.

1. For all $n \in \mathbb{N}$, $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$.

2. For all $n \in \mathbb{N}$, $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

3. Let $x > -1$ be a real number. Then for all $n \in \mathbb{N}$, $(1+x)^n \geq 1+nx$.

4. Let A be a finite set of size n . Then $|\mathcal{P}(A)| = 2^n$.

(1) For all $n \in \mathbb{N}$, $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$. Then use this to prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots = 1$$

6. For all $n \in \mathbb{N}$, $\frac{n(n+1)(n+2)}{1 \cdot 2 \cdot 3}$ is an integer. Then show that for all $n \in \mathbb{N}$, $\frac{n(n+1)(n+2)(n+3)}{1 \cdot 2 \cdot 3 \cdot 4}$ is an integer.

7. For all $n \in \mathbb{N}$, $3 \mid 2^n + 1 \iff 3 \nmid 2^n - 1$.