Math 21b, April 25: Some more PDEs and Review

1. Find the solution f(x,t) to the inhomogeneous partial differential equation

$$f_t = -f_{xx} + \sin(7t)$$

given the initial condition f(x, 0) = x. Use the following steps:

- Find the general form of the solution to the *homogeneous* equation using Fourier series in x.
- Find a particular solution to the above inhomogeneous equation using the cookbook method. Trick: find a solution to $f_t = \sin(7t)$, and the just use this particular solution!
- Add these two to get the general form of the solution for the inhomogeneous equation.
- Solve for the initial Fourier coefficients by finding a sine series for f(x, 0) = x.

2. (HW31 #4) A laundry line is excited by the wind. It satisfies the differential equation

$$u_{tt} = u_{xx} + \cos(t) + \cos(3t)$$

with initial conditions $u(x, 0) = 4\sin(5x) + 10\sin(6x)$ and $u_t(x, 0) = 0$. Find the function u(x, t) which satisfies the differential equation.