## Second Order Linear Differential Equations

1. Warm-up: What were the eigenvalues and associated eigenfunctions of $D$ on $C^{\infty}$ ?
2. Set $C_{\text {per }}^{\infty}$ to be the set of all smooth $f$ that have period $2 \pi($ so $f(t)=f(t+2 \pi))$.
(a) Which of the eigenfunctions from $\# 1$ are periodic? What are their periods?
(b) When are the eigenfunctions above in $C_{p e r}^{\infty}$ ?
(c) What are the eigenvalues and associated eigenfunctions of $D$ on $C^{\infty}$ ?
3. Define $S_{\lambda}$ to be the function on $C^{\infty}$ given by

$$
S_{\lambda} f(t)=C e^{\lambda t}+e^{\lambda t} \int_{0}^{t} e^{-\lambda s} f(s)
$$

Show that $y=S_{\lambda} f(t)$ solves $(D-\lambda) y=f(t)$.
4. Find the general solution for

$$
y^{\prime \prime}-2 y^{\prime}+y=0
$$

5. Find the general solution for

$$
y^{\prime \prime}-2 y^{\prime}+y=\sin 2 t
$$

(a) First, using the cookbook method.
(b) Second, by using the right inverse of $D-\lambda$.
6. Find the general solution for

$$
y^{\prime \prime}-3 y^{\prime}+4 y=0 ; y(0)=1, y^{\prime}(0)=3
$$

7. Find the general solution for

$$
f^{\prime \prime}+f=\cos t
$$

8. Find the general solution for

$$
f^{\prime \prime}+f^{\prime}-6 f=e^{t}+t
$$

