Second Order Linear Differential Equations

1. Warm-up: What were the eigenvalues and associated eigenfunctions of D on C^{∞} ?

2. Set C_{per}^{∞} to be the set of all smooth f that have period 2π (so $f(t) = f(t + 2\pi)$).

(a) Which of the eigenfunctions from #1 are periodic? What are their periods?

(b) When are the eigenfunctions above in C_{per}^{∞} ?

(c) What are the eigenvalues and associated eigenfunctions of D on C^{∞} ?

3. Define S_{λ} to be the function on C^{∞} given by

$$S_{\lambda}f(t) = Ce^{\lambda t} + e^{\lambda t} \int_{0}^{t} e^{-\lambda s} f(s)$$

Show that $y = S_{\lambda}f(t)$ solves $(D - \lambda)y = f(t)$.

4. Find the general solution for

$$y'' - 2y' + y = 0$$

5. Find the general solution for

$$y'' - 2y' + y = \sin 2t$$

(a) First, using the cookbook method.

(b) Second, by using the right inverse of $D - \lambda$.

6. Find the general solution for

$$y'' - 3y' + 4y = 0; y(0) = 1, y'(0) = 3$$

7. Find the general solution for

$$f'' + f = \cos t$$

8. Find the general solution for

$$f'' + f' - 6f = e^t + t$$